And the entire moment when moving with proper angular velocity is

$$\int_{\frac{\pi}{2}}^{z_1} -\tfrac{8}{9} fv^2\pi r^3 \tan^3 z_1 \frac{\cos^2 z dz}{\sin^3 z} = \tfrac{4}{9}\pi fv^2 r^3 \tan^3 z_1 \left(\frac{\cos z_1}{\sin^2 z_1} + \log\tan\tfrac{1}{2}z_1\right).$$

Hence, $\dfrac{\frac{4}{9}fv^2 r \tan z_1[(\cos z_1 \div \sin^2 z_1) + \log\tan\frac{1}{2}z_1]}{(\cos z_1 \div \sin^2 z_1) - \log\tan\frac{1}{2}z_1}$ = a maximum for z_1, (2)

will give the position of wings when the work being done is a maximum.

Therefore, placing the first differential coefficient = 0, and solving for $\log\tan\frac{1}{2}z_1$, there results

$$\log\tan\tfrac{1}{2}z_1 = (\cos z_1 \div \sin^2 z_1)[1 + \cos^2 z_1 \pm \sqrt{(4 + \cos^4 z_1)}].$$

Substituting in (2) this second value, the only one applicable,

$$\frac{\tan z_1[1 + 1 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)}]}{1 - 1 - \cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}} = \frac{\tan z_1[2 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)}}{\sqrt{(4 + \cos^4 z_1)} - \cos^2 z_1}$$

$$= -\tan z_1 + \tfrac{1}{2}\tan z_1[\cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}] = \text{maximum for } z_1.$$

Differentiating a second time and reducing,

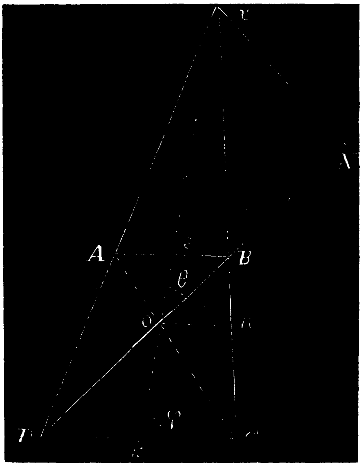
$$\cos^6 z_1 - \tfrac{7}{2}\cos^4 z_1 - 2\cos^2 z_1 + 2 = 0, \text{ from which}$$

$$\cos^2 z_1 = 0.55154\,;\ \therefore z_1 = 42°2\tfrac{1}{2}'.$$

SOLUTION OF A PROBLEM.

BY PROF. C. M. WOODWARD, WASHINGTON UNIV., ST. LOUIS, MO.

Problem.—Given a frustrum of an oblique cone with a circular base; the frustrum is cut in two by a plane perpendicular to the principal plane of the cone, and tangent to the two bases. Find the ratio of the volumes of the two parts of the frustrum.



Solution.—Let $vABCD$ be the section of the cone made by the principal plane, and let DBN be the trace of the intersecting plane perpendicular to it. The section, projected in DB, is obviously an ellipse of which DB is the major axis. Let $ov = h$, $As = r$, $Dk = R$ and $on = x$. Then

$$DB = (R + r)\frac{\sin\varphi}{\sin\theta}.$$

The minor axis is the chord of a circle whose radius is $\frac{1}{2}(R + r)$, and whose dist. from the center is $\frac{1}{2}(R - r)$; its length is therefore $2\sqrt{(Rr)}$.

The volume of the cone $v—DB$ is

$$V_0 = \pi \frac{R+r}{2} \cdot \frac{\sin\varphi}{\sin\theta}\left(R\,r\right)^{\frac{1}{2}} \frac{h\sin\theta}{3} = \tfrac{1}{3}\pi h \sin\varphi\, \frac{R+r}{2}\left(R\,r\right)^{\frac{1}{2}},$$

but $$\frac{R+r}{2} = \frac{R\,r}{x}; \text{ hence } V_0 = \frac{1}{3x}\pi h \sin\varphi\, R^{\frac{3}{2}} r^{\frac{3}{2}}.$$

Now $$vs = \frac{hr}{x} \text{ and } vk = \frac{h\,R}{x},$$ hence

cone $v—AB = V_1 = \dfrac{1}{3x}\pi h \sin\varphi\, r^3$, cone $v—CD = V_2 = \dfrac{1}{3x}\pi h \sin\varphi\, R_3$.

$$\text{Vol. } ABD = V_0 - V_1 = \frac{1}{3x}\pi h \sin\varphi\left(R^{\frac{3}{2}} - r^{\frac{3}{2}}\right) r^{\frac{3}{2}},$$

$$\text{Vol. } BDC = V_2 - V_0 = \frac{1}{3x}\pi h \sin\varphi\left(R^{\frac{3}{2}} - r^{\frac{3}{2}}\right) R^{\frac{3}{2}},$$

hence $$\frac{\text{Vol. } ABD}{\text{Vol. } BDC} = \frac{\sqrt{r^3}}{\sqrt{R^3}}.$$

From the volumes of the three cones we see that the elliptical cone is a mean proportional between the other two. (This striking analogy between the cones $v — AB$, $v — BD$, $v — CD$, and the triangles AvB, DvB, DvC, I had never before noticed, though it must have been well known. Neither had I ever observed that the semi-conjugute axis of *such* a conic section is a mean proportional between the radii of the bases of the frustrum.)

It is obvious that the plane AC divides the frustrum in the same ratio as the plane BD.

If the altitude of the frustrum is p we have $h\sin\varphi \div x = p \div (R—r)$ and

$$V_0 = \tfrac{1}{3}\pi p[\sqrt{(R^3 r^3)} \div (R—r)],$$

$$V_1 = \tfrac{1}{3}\pi p[r^3 \div (R—r)], \quad V_2 = \tfrac{1}{3}\pi p[R^3 \div (R—r)],$$

$$V_0 — V_1 = \tfrac{1}{3}\pi p \frac{\sqrt{R^3} — \sqrt{r^3}}{R—r}\, r^{\frac{3}{2}},$$

$$V_2 — V_0 = \tfrac{1}{3}\pi p \frac{\sqrt{R^3} — \sqrt{r^3}}{R—r}\, R^{\frac{3}{2}}.$$

SOLUTION OF PROBLEM 255.

BY PROF. W. W. HENDRICKSON, NAVAL ACADEMY, ANNAPOLIS, MD.

DENOTING the distance AB by a, and taking the axes as represented in the figure, the equations to the lines AC and BC are (1.) $y = x\tan(\varphi + a)$,